Preservation Value in SES

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I-Site Clermont Seminar
IRSTEA, Aubière, October 5, 2018

Context

- Capturing sustainability is challenge due to the many connections and feedbacks between and within social and ecology systems. One tool to capture these connections are socio-ecological system (SES) models.
- Those have proved to be a useful organizing device to capture these links and feedbacks between humans and their environment (Ostrom, 2009).
- A typical SES is composed of anthropogenic and natural elements interacting through temporal, spatial, and organizational scales.
- The organizational scale is composed of nodes, such as natural components, resource users, civil players, voters, economic actors or regulatory organizations, and of linkages between those nodes, like exchanges or transfers of money, energy, information and strategies.

Context

- The key to extracting useful information from these SES models is to address the degree and level of connectivity, which we define as the property of all types of elements interacting on a network.
- The analysis of SES sustainability has been mostly conducted through the idea of resilience (Gonzalès and Parrott, 2012). A system is considered to be resilient when it adapts to external perturbations while continuing to function.
- Researchers and policymakers recognize that in reality risks exist within an SES system that can work to undercut the connectivity, which will undermine the goal sustainability. These risks can be both intra- and inter-layer disconnections.
- Small research agenda has been devoted to the economic value of connectivity (Dragicevic et al., 2017 ; Dragicevic, 2018).

Context

- We develop a model that reveals the preservation value of maintaining connectivity within an SES system.
- We define two measures of preservation value of inter- and intra-layer connections.
- We show under which conditions connectivity is valuable and should be preserved.
- Our results suggest that the preservation value of the SES topological structure is greatest when we secure the connectivity of inter-layer connections.

Model

- Consider an undirected and unweighted multiplex network, based on the Euclidean metric, of dimension $\mathbb{R}^N$.

- The network is represented by an undirected graph $\Gamma = \{V, E\}$, which consists of vertices $V = \{1, ..., N\}$ and of the set of edges $E = \{(i, j) \in V \times V\}$, which represent the inter-node interactions.

- The population of nodes is distributed among L_n layers, where $n = 1, 2, 3$. Each layer contains N_n nodes, with $i_n = 1, ..., N_n$, with different intra-layer connectivity.

Model

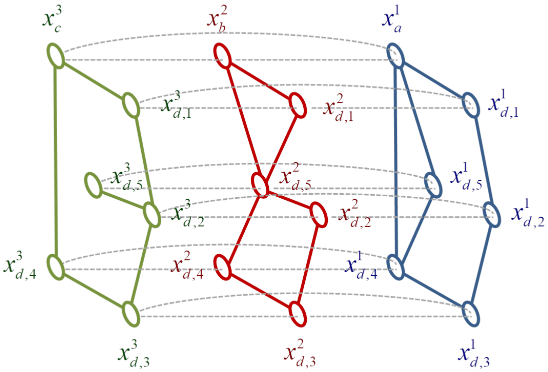

FIGURE: The SES multiplex graph, inspired by the sustainability Venn diagram, is composed of economic (blue), social (red) and environmental (green) layers. Each of them is composed of six connected nodes. A set of thirty-eight edges, which can be either intra- or inter-layer connections, forms the SES multilayered network. Spatial discount factor δ^l weighs up the distance between two nodes.

Model

- Such a multiplex system is completely specified by the vector of the adjacency matrices of the n layers. Let A^n, for $n = 1, 2, 3$, be the adjacency matrix of L_n with nonnegative elements $(a_{ij}^n)_{N \times N}$, for $i_n = 1, ..., N_n$.

- Assume the existence of a convex hull of vertices Ω to be an N-simplex, with the Euclidean norm in $\mathbb{R}^N$.

- Let $x_i^n(e_i^n, t) \in \mathbb{R}^N$, where $n = 1, 2, 3$ and $i_n = 1, ..., N_n$, denote the state of node i_n, characterized by its feature e_i^n at time t.

Model

- Let Λ be the set of nodes, such that the nodes connected to node $i \in \Lambda$ are referred as to subset Λ^i. For $\forall i, j \in \Lambda$, $d_{ij} = |x_i(e_i) - x_j(e_j)|$, and for $\Lambda^i = \{j \in \Lambda : 0 < d_{ij} \leq z\}$, d_{ij} and z respectively stand for the Euclidean distance between nodes, and their respective interaction capacity.

- Nodes obtain and provide utility u_{ij} from and to other nodes.

Definition 1

For $\forall i, j \in \Lambda$, $d_{ij} = |x_i(e_i) - x_j(e_j)|$, and for $\Lambda^i = \{j \in \Lambda : 0 < d_{ij} \leq z\}$,

$$u_{ij} \quad \begin{cases} > 0 & \text{if } a_{ij} > 0 \\ = 0 & \text{otherwise} \end{cases}$$

- We assume that only connections provide utility, knowing that the nodes predate the network construction.

Model

- The Euclidean distance becomes irrelevant if the interaction occurs between nodes defined as incompatible. In other words, a node sufficiently close, but endowed with a different feature, cannot provide any utility to the interacting node.

- For that reason, let us introduce the Mahalanobis distance. As such, smaller distances correspond to interacting nodes that are designated as similar or compatible (Dragicevic et al., 2017).

- Following Shaw et al. (2011), consider an Euclidean distance metric parameterized by a positive semidefinite matrix $\Pi = L^T L \equiv S^{-1}$, where $\Pi \in \mathbb{R}^{N \times N}$ and $L \in \mathbb{R}^{N \times N}$.

Model

- The three-dimensional Mahalanobis distance $m(C)_{abc}$ between nodes $a \in \Lambda_1$, $b \in \Lambda_2$ and $c \in \Lambda_3$, issued from the three layers, corresponds to

$$m(C)_{abc} = \left[\left(x_a^1(e_a^1) - \sum_{l=0}^{L} \delta^l \left(x_b^2(e_b^2) + x_c^3(e_c^3) \right) \right)^T S^{-1} \left(x_a^1(e_a^1) - \sum_{l=0}^{L} \delta^l \left(x_b^2(e_b^2) + x_c^3(e_c^3) \right) \right) \right]^{\frac{1}{2}} C \qquad (1)$$

  where $\sum_{l=0}^{L} \delta^l$ denotes the composite factor of the spatial discounting dependent on the sequence of vertices the distances of which are being measured.

- The nodes specified as compatible shall be linked, which is verified by $S^{-1} \geq 0$.

- The Mahalanobis distance metric guarantees that the connection occurs when two nodes display compatible characteristics.

Model

- In light of finiteness of resources, nodes that interact build a grid dependent of their opportunity costs. Let scalar C be this economic opportunity cost, from choosing either node from the multiplex graph, computable at the market value.

- A network administrator is able to identify the subset of nodes Λ_n, for $n = 1, 2, 3$, evolving on either layer through intra-layer connections, which all have counterparts on other layers.

- The number of nodes in each subset is respectively given by $|\Lambda_1| = N_1$, $|\Lambda_2| = N_2$ and $|\Lambda_3| = N_3$.

- As for subsets Λ_4 and Λ_5, they correspond to regions delimiting inter-layer connections.

Model

- Following Gustavi et al. (2010), the follower node dynamics is given by the Laplacian-based control strategy (consensus) differential equation, meaning that the state of a node evolves according to the states of the nodes to which it is connected.

- The dynamics for a node can be written as

$$\dot{x}_a^1(e_a^1) = -Nx_a^1(e_a^1) + \frac{\delta^{L+1} - 1}{\delta - 1} \left[N \left(x_b^2(e_b^2) + x_c^3(e_c^3) \right) - \left(x_d^1(e_d^1) + x_d^2(e_d^2) + x_d^3(e_d^3) \right) \right] \quad (2)$$

Lemma 1

The network equilibrium under consensus dynamics corresponds to the annulment of differential equation $\dot{x}_a^1(e_a^1)$ weighted by the spatial discount factor up to the graph diameter.

Model

Theorem 1

Given the consensus problem is well-defined in the initial state, the SES equilibrium is at a steady state when the whole is greater than the sum of its parts (i.e., formally, when the marginal variation of the SES barycenter exceeds the marginal variation of a node's utility set).

- The result shows that, for SES to remain in equilibrium, its center of gravity needs to be greater than the aggregate of states of nodes connecting the layers with respect to the graph diameter or in the longest path.
- Let us now derive general conditions for the layers to remain connected.

Model

- The connectivity relation $\dot{m}(C)_{abc}$ defines the preservation of the network connectedness. The condition for nodes i and j to evolve connected is $\dot{m}(C)_{ij} \leq 0$.

- For arbitrary nodes $a \in \Lambda_1$, $b \in \Lambda_2$ and $c \in \Lambda_3$, the connectivity is defined by

$$
\begin{aligned}
\dot{m}(C)^2_{abc} \quad = \quad & 2N \left[\left(x^1_a(e^1_a) - \sum_{l=0}^{L} \delta^l \left(x^2_b(e^2_b) + x^3_c(e^3_c) \right) \right)^T S^{-1} \left[-x^1_a(e^1_a) \left(2 \frac{\delta^{L+1} - 1}{\delta - 1} + 1 \right) \right] \right] C^2 \\
+ \quad & 2N \left[\left(x^1_a(e^1_a) - \sum_{l=0}^{L} \delta^l \left(x^2_b(e^2_b) + x^3_c(e^3_c) \right) \right)^T S^{-1} \left[x^2_b(e^2_b) + x^3_c(e^3_c) \right] \right] C^2 \\
- \quad & 6 \left[\left(x^1_a(e^1_a) - \sum_{l=0}^{L} \delta^l \left(x^2_b(e^2_b) + x^3_c(e^3_c) \right) \right)^T S^{-1} \left[\frac{\delta^{L+1} - 1}{\delta - 1} \left(x^1_d(e^1_d) + x^2_d(e^2_d) + x^3_d(e^3_d) \right) \right] \right] C^2
\end{aligned}
\tag{3}
$$

Corollary 1

Necessary and sufficient condition for arbitrary nodes from different layers to evolve connected in a multilayered graph is that their states be similar.

Model

- Following the methodology by Mesbahi and Egerstedt (2010), let us introduce the performance function J, which measures the preservation of the network weighted by the opportunity costs.

$$J = \int_0^T \sum m(C)_{abc} \, dt \tag{4}$$

- The network administrator decides to sustain the network topology via control variables $m(C)_{abc}$. Put differently

$$\min_{x_a^1(e_a^1),\, x_d^1(e_d^1)} J \tag{5}$$

subject to two first-order dynamic constraints

$$\dot{x}_a^1(e_a^1), \; \dot{x}_d^1(e_d^1) \tag{6}$$

Model

- The optimal control problem is solved by means of the present value Hamiltonian, discounted in time up to $t = T$, which represents the impact of evolution of $x_a^1(e_a^1)$ and $x_d^1(e_d^1)$ on the network topology. The first-order optimality conditions yield

$$\lambda = \mu \frac{\delta + 2\delta^{L+1} - 3}{2\delta(1 - \delta^L)} \tag{7}$$

Lemma 2

The preservation of SES inter- and intra-layer connections is subject to imperfect strategic substitutability.

- The expression being incalculable for $\delta = \{0, 1\}$, we are in presence of imperfect strategic substitutes.

Model

- By letting $w = \left[x_a^1(e_a^1)^T, x_b^2(e_b^2)^T, x_c^3(e_c^3)^T, x_d^1(e_d^1)^T, x_d^2(e_d^2)^T, x_d^3(e_d^3)^T, \lambda^T, \mu^T\right]^T$ reflect the network state, the system control is obtained through the following Hamiltonian system.

$$\dot{w} = Pw \tag{8}$$

where P is a matrix built upon the first-order and boundary conditions.

Theorem 2

The SES multiplex state w evolves optimally according to coordinates P.

Simulations

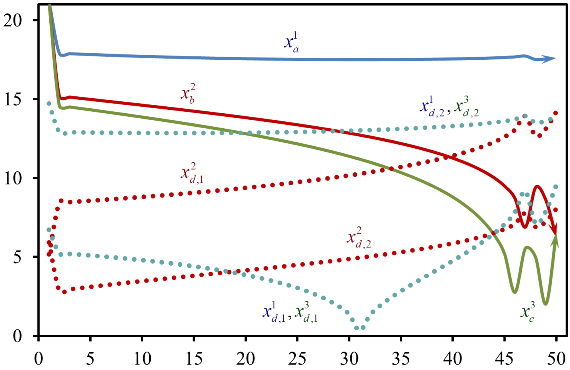

FIGURE: Mahalanobis coordinates (ordinates), spatially discounted at $\delta^l = 0.02$, as functions of time (abscissa) of three inter-layer connected nodes, coupled with six additional nodes which constitute the intra-layer connections.

Simulations

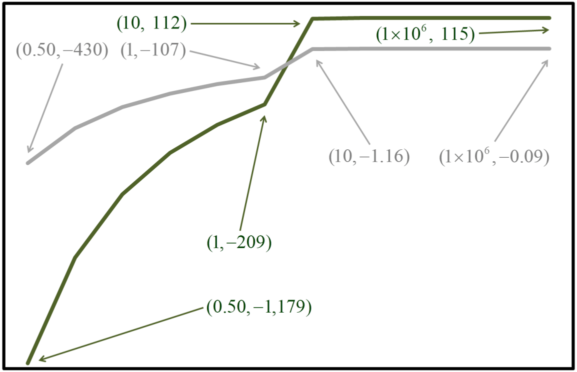

FIGURE: SES preservation through its inter-layer connections (SES(λ)). The green curve depicts the preservation values of inter-layer connections issued from inter-layer control ($\lambda_{\lambda(P)} \Rightarrow$ SES(λ)). The grey curve represents the preservation values of intra-layer connections issued from inter-layer control ($\mu_{\lambda(P)} \Rightarrow$ SES(λ)).

Simulations

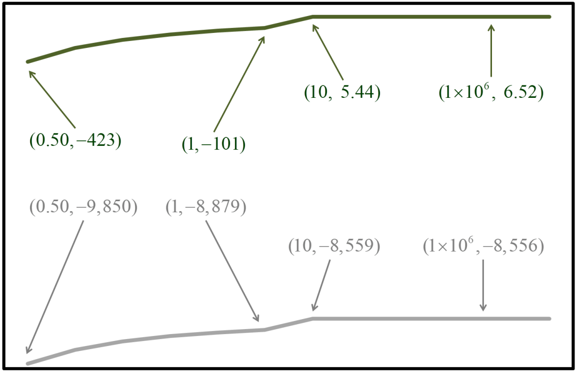

FIGURE: SES preservation through its intra-layer connections (SES(μ)). The green curve depicts the preservation values of inter-layer connections issued from intra-layer control ($\lambda_{\mu(P)} \Rightarrow$ SES(μ)). The grey curve represents the preservation values of intra-layer connections issued from intra-layer control ($\mu_{\mu(P)} \Rightarrow$ SES(μ)).

Simulations

Result 1

The shadow prices are negatively unbounded at near-zero opportunity costs and positively bounded with significant levels of opportunity costs.

Result 2

Preserving the SES structure through optimal control is more efficient by securing connectivity of inter-layer connections than of intra-layer connections, because substitutability is more acute in case of intra-layer control.

Discussion

- We could think of the relevance to preserve the SES structure through its inter-layer connections as our main result.
- The works by Mäler (2008) and Mäler and Li (2010) dealt with resilience management as insurance against reaching a non-desired state. Our results are partly in accordance with their findings.
- Baumgärtner and Strunz (2014) studied the ecosystem resilience as an insurance tool that enables to mitigate against potential welfare losses. Measuring the preservation of connectivity in time boils down to setting apart a component of this value.
- A geographically-based cost structure to forming links was presented in Jackson and Rogers (2005). This work extends their findings : (1) presence of characteristics of a multiplex small world network (Agarwal et al., 2016) ; (2) instead, a higher (shadow) cost is meant to avoid severing links.

Thank you for your attention